Notes on Phase Transitions in Multiagent Systems

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One can search for a solution to a wicked problem, but a wicked problem (especially in multi-agent systems such as human societies) is a nonlinear system of wicked problems and therefore cannot be solved but only managed. Or one can look for an ordered sequence of non-equilibrium phase transitions leading towards less and less "wicked" states of the problem. Game theory and Physics provide many mathematical tools to study non-equilibrium phase transitions. These seem to show that cooperation states are precarious but also that collective learning, voluntary participation and forms of entanglement are possible cooperation "boosts".

1. Prelude: Correlated Equilibria

In 1949 John Nash proved that any competitive game has at least one kind of equilibrium, i.e. a stable and optimal state of interactions among the players in which you don't gain anything by changing the way you are playing as long as the others keep playing the way they are playing. Therefore there is no motivation to change the way each player is playing and the game will continue forever with each player taking the same action forever (if the players are rational, of course). When there are only two "players", the point of equilibrium is not difficult to determine, but, when there are thousands of "players" (as in many social situations), there are an almost infinite ways that the game could play out, possibly more ways than time available in the future of the universe. Economists often use the concept of Nash equilibrium to justify economic policies. In reality, Nash's theorem shows that is always an equilibrium but it doesn't show how to figure out what the equilibrium is. In fact, in 2016 Yakov Babichenko and Aviad Rubinstein showed that a general procedure for players to find a Nash equilibrium does not exist ("Communication Complexity of Approximate Nash Equilibria"). There are two known ways out of it. First, the players can communicate to each other everything about their respective preferences; but of course this becomes combinatorially prohibitive as the number of players increases. Second, Robert Aumann's "correlated equilibrium" ("Subjectivity and Correlation in Randomized Strategies", 1973): each player is advised by a "correlating device" (a trusted mediator) about what strategy to play. For example, the correlating device in city traffic is the traffic signal that decides which car can go first through an intersection. Aumann's correlated equilibria are not necessarily Nash equilibria. There are two advantages in using correlated equilibria: 1. in some circumstances they result in more positive outcomes than Nash equilibria; 2. the equations of correlated equilibria are more friendly than the equations of Nash equilibria.

2. Collective Learning in Multiagent Systems

Yuzuru Sato and Jim Crutchfield at the Santa Fe Institute worked out equations that describe the dynamics of collective learning in multiagent systems ("Coupled Replicator Equations for the Dynamics of Learning in Multiagent Systems", 2003). These equations are related to replicator dynamics that was introduced by Peter Taylor and Leo Jonker for evolutionary game theory ("Evolutionarily Stable Strategies and Game Dynamics", 1978) as an extension of John Maynard Smith's and George Price's "evolutionarily stable strategy" discussed in their article "The Logic of Animal Conflict" (1973) to describe a Nash equilibrium that is stable over the evolution of a population of interacting agents. The equations of replicator dynamics were shown to be related to reinforcement learning by Tilman Boergers and Rajiv Sarin ("Learning through Reinforcement and Replicator Dynamics", 1997). Sato and Crutchfield adapted this result to collective learning.

3. Unstable Equilibria for Cooperation

In 2014 Joshua Plotkin and Alexander Stewart at the University of Pennsylvania discovered a very general model for when cooperation is expected, or not expected, to evolve in a group. Their model allows each member of the group to play a series of games (the iterated Prisoner's Dilemma) with every other member of the group ("Collapse of Cooperation in Evolving Games", 2014). The result is that cooperation can easily collapse into competition just by tweaking a variable.

4. Games and Phase Transitions

A phase transition can be viewed as due to the competition between two opposing orders. For example, Tieyan Si of the Chinese Academy of Sciences in Beijing argued that a phase transition is a war game ("Game Theory and Topological Phase Transition", 2008). He conceived the boundary between the old phase and the new phase as the outcome of many rounds of negotiation between the old order and the new order.

5. Non-equilibrium Phase Transitions

"Equilibrium" refers to a state in which an object is not in motion and has no energy flowing through it. The vast majority of natural phenomena (such as the lightning strike of a thunderstorm) and human phenomena (pretty much everything that happens in a society) occur far from equilibrium. In particular, humans in their societies are constantly acting and causing others to act. Nonetheless, Physics has traditionally studied phase transitions only as states of equilibrium. Haye Hinrichsen's lecture "Non-equilibrium Phase Transitions" (2005) lamented that "the experimental evidence is still very poor".

Christoph Hauert of the University of British Columbia (https://www.math.ubc.ca/~hauert/) wrote papers such as "Phase Transitions and Volunteering in Spatial Public Goods Games" (2002) and "Game Theory and Physics" (2004), in collaboration with Gyorgy Szabo of the Research Institute for Technical Physics and Materials Science in Hungary, that presented a mechanism for sustainable cooperation: voluntary participation, or, better, voluntary public-goods interactions. The "public-goods game" is a simple model of cooperation in which the participants receive a fixed amount of money at the start of every round and can either place it in a communal pot or keep it, with different rewards for each behavior. "Voluntary" refers to the additional policy of allowing risk-averse players to drop out of the game and become "loners". It turns out that this additional rule does not lead to equilibrium but to a series of phase transitions: from a state in which loners (opposed to cooperation) dominate to a state in which loners coexist with cooperators and defectors, to a state in which cooperators form traveling waves that are attacked on one side by new defectors and strengthened on the other by converted loners, and finally to a state with no loners and with large clusters of cooperators surrounded by clouds of defectors. In the end, this game makes cooperation more likely to persist among the ones don't opt out. The phase transitions and traveling waves are similar to the ones studied by solid-state physics. Another route to dealing with non-equilibrium phase transitions is to view them as Moebius transformations. In 2017 Valerii Vinokur and Alexey Galda at the Argonne National Laboratory proposed a way to mathematically describe a phase transition in a system far from equilibrium. They described the system in quantum equations using its energy (as usual) and adding an imaginary force that accounts for the deviation from equilibrium; a mathematically "imaginary" number ("Linear Dynamics of Classical Spin as Moebius Transformation", 2017).

6. Quantum Games

Game theory provides a quantitative framework for analyzing the behavior of rational agents, but usually assumes that the action of the players don't influence each other. Quantum games are games in which the choices of the players are "entangled" so that each influences the other. The theory of quantum games combines classical game theory and quantum information theory. Jens Eisert and Matrin Wilkens at the University of Potsdam investigated the quantization of the famous game of Prisoners' Dilemma and introduced an elegant method for quantizing games ("Games and Quantum Strategies", 1999). They found that the Prisoners' Dilemma is no longer a dilemma, and others that came later showed that quantum games can outperform their classical counterparts. Notably, Simon Benjamin and Patrick Hayden of Oxford University showed that multiplayer quantum games can exhibit forms of quantum equilibrium that have no analogue in classical games ("Multi-player Quantum Games", 2000). See this brief summary by Phil Ball in Nature (1999): http://www.nature.com/news/1999/991018/full/news991021-3.html

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"Active matter" physics, a field jump-started by Tamas Vicsek in Hungary ("Novel Type of Phase Transition in a System of Self-Driven Particle", 1995), studies how simple mathematical rules governing interactions between individuals can give rise to large-scale order. Active-matter physics has successfully explained phase transitions such as how water molecules crystallize into ice and how atomic spins align to form magnets. In 2017 Hong Kong scientists of Yilin Wu's lab showed that even the behavior of bacteria can be explained as active matter: what looks like the random motions of individual bacteria can turn into synchronized oscillations at large scales ("Weak Synchronization and Large-scale Collective Oscillation in Dense Bacterial Suspensions", 2017).